Conditions under which a superdiffusive random-search strategy is necessary

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Intuitively, lower target densities and lower detection capabilities should demand more sophisticated search strategies for a random search reasonable outcome. In contrast, when targets are easily found, a simple Brownian random walk strategy is enough. But where is the threshold between these two scenarios and when is optimization really necessary? We address this considering the interplay between two essential scales in random search, the average distance between neighbor targets \( l_0 \) and the detection capability \( r_v \). In the limit cases the ratio \( \beta = r_v/l_0 \) suffices to characterize the problem. For low (high) \( \beta \) a superdiffusive behavior is (is not) crucial for the process optimization. However, there is a crossover range, which is a nontrivial function of \( r_v \) and \( l_0 \), separating the two regimes. We analyze this intermediate region, common in nature, and discuss the often overlooked important trade between resources availability and the searcher location power. Our results highlight contexts where efficient random search is a key factor for survival, such as in animal foraging.

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I. INTRODUCTION

Random search (RS) [1] has applications as diverse as human mobility [2] and regulatory proteins “looking for” DNA spots [3]. But it is animal foraging with incomplete information on the targets that RS has been more intensively studied [4,5]. In RS important questions include (1) how different environmental factors influence efficiency and (2) when the choice of strategy becomes fundamental for survival. To understand such issues, aspects as landscape heterogeneity [6], memory effects [7], intermittent dynamics [8,9], target regeneration [8,10], and energy balance [11] have been analyzed. However, an important and unexplored key point is the interplay between the searcher’s perceptual range (detection radius \( r_v \)) and the target density \( \rho \) (in \( nD \) parameterized via a length scale \( l_0 \), or \( \rho \equiv l_0^{-n} \)). \( \beta = r_v/l_0 \) should significantly impact the behavior of optimal search, but precisely how? For instance, could increasing \( l_0 \) by a certain factor (diminishing targets availability) be compensated by increasing \( r_v \) by the same factor? These are fundamental queries to determine the optimal strategy for a RS process.

Here we address the above assuming Lévy walks RS [4]. Although minimalistic, such a framework is very general, allowing one to infer, e.g., detection power, targets density, and searcher diffusiveness, in a rather direct way (in opposition to many dedicated models from which extended results cannot be drawn). So Lévy walks are quite appropriate to discuss broad problems in RS processes [6].

We report in the two-dimensional (2D) case (relevant and realistic in many instances) a crossover region separating two regimes: optimal strategy given by super- or normal diffusion. We find that efficiency and mean number of steps between consecutive encounters present scale-invariant behavior for low \( \beta \), for which optimization is crucial. Also, we unveil the influence of the searcher perceptual range and target density to the emergence of optimal search in the relevant transition region, where strategy-switching mechanisms would be in order to improve the search performance in a changing environment.

II. MODEL AND DYNAMICS

The landscape is constituted by \( N \) randomly distributed targets in an area \( L^2 \), with periodic boundary conditions (to facilitate statistical averages), so \( \rho \equiv l_0^{-n} = N/L^2 \). Hence, \( l_0 \) is a characteristic length proportional to the mean distance between neighbor targets (Fig. 1). Revisits to a same target are allowed [12]. The searcher follows two rules [10] (see Fig. 1):

1. If there are targets located within the searcher perceptual (“direct vision”) distance \( r_v \), it moves on a straight line to the nearest target.
2. If there is no target within a \( r_v \), in its \( j \text{-th} \) step the searcher chooses a random direction and a distance \( s_j \geq r_v \) from a probability density function (pdf) \( p_j(s_j) = p(s_j) \). It then moves to the new point, continually looking for a target within \( r_v \) along the way. If it does not detect a target, it stops after traversing the distance \( s_j \) and chooses a new direction and a new distance \( s_{j+1} \). Otherwise, it truncates the step upon encountering the target and proceeds as in rule (1).

We assume a truncated Lévy distribution [13], or

\[
p(s_j) = \begin{cases} \frac{(\mu - 1)/(r_v^{1-\mu} - l^{1-\mu})}{s_j^{-\mu}}, & r_v < s_j \leq l \\ 0, & \text{otherwise.} \end{cases}
\]

(1)

Here \( l \) is a cutoff length associated with the system natural bound, usually with \( l \gg l_0 \). For \( l \rightarrow \infty \), \( p(s_j) \) represents the long-range asymptotical behavior of the \( \alpha \)-stable Lévy distributions with \( \alpha = \mu - 1 \) [14]. The second moment diverges for \( 1 < \mu \leq 3 \) and superdiffusion takes place [14]. The mean-squared displacement of a random walker scales with time as \( t^{2H} \), for \( H \) the Hurst exponent. For Lévy walks and superdiffusion \( H > 1/2 \) [14], whereas diffusive Brownian behavior \( (H = 1/2) \) emerges for \( \mu > 3 \). For finite \( l \), \( p(s_j) \) tends to normal diffusion [13] after very long times. For the
respectively. Associated to landscapes very rich and very scarce in targets, occur much more frequently. These limiting regimes can be given position to look randomly for targets, rule (2), should the searcher has a reduced detection capability. Leaving a

The mean number of reorientations between successive encounters is the inverse of the mean single-step length \[1, 16\]. The distance travelled between encounters and the average distance traveled between targets is quantified by \[\ell\] and \[\rho\], respectively. All quantities next are obtained by averaging over \[N_r\] runs (a search process that continues until finding \[N_t\] targets). We discuss extensive numerical simulations, showing representative examples for \[L = 10^4\], \[l = 5L\], \[N_r = 10^5\], \[N_t = 500\], \[5 \times 10^{-3} \leq \beta \leq 1\], and \[1 \leq \rho \leq 10\]. Thus, \[2.5 \times 10^{-5} / \rho^2 \leq \rho \leq 1 / \rho^2\], and \[\rho \leq 10 \leq 200 \rho\].

Figure 2 displays \[\eta\] versus \[\rho\], for several \[\beta\] and \[\mu = 2.0\] (superdiffusive) and \[\mu = 3.0\] (diffusive) behavior. We see that \[\beta\] significantly impacts the best strategy. For \[\beta \lesssim 0.2\], superdiffusive searches are increasingly more efficient than diffusive ones. For \[\beta = 0.1\], \[\eta(\mu = 2)\] is about 7% higher than \[\eta(\mu = 3)\] for either \[\rho = 10\] or \[1\]. For \[\beta = 0.007\] the difference increases to 15% for \[\rho = 10\] and 21% for \[\rho = 1\]. The optimal \[\mu \approx 2\] strategy generally arises in scarce environments, in which a large number of small steps alternate with a few, but relevant, large steps, allowing to access previously unvisited regions \[10\]. For \[\beta > 0.2\], normal diffusion is better than the \[\mu = 2.0\] dynamics, with standard Brownian walks already yielding very reasonable outcomes.

In Fig. 2, \[\eta\] decays as a power law of \[\rho\]. Hence, as \[\beta\] is kept fixed, the efficiency gain due to an increasing in the perceptual range \[\rho\] does not compensate the efficiency lost associated with the corresponding decreasing in density (recall \[\rho = l_0^{-2} = (\beta / \rho_v)^2\]). This leads to a global lowering of \[\eta\] in scarcer landscapes. Another asymmetry between \[l_0\] and \[\rho\] comes from \[p(s_j)\] lower cutoff, which depends only on \[\rho\]. It is not an model artifact, however, since in a RS with low information it is realistic to suppose a minimum step length function of the local detection range but not of the global \[\rho\].

Figure 3 displays a narrow crossover region separating the normal, \[\mu_{opt} \approx 3\], and superdiffusive, \[\mu_{opt} \approx 2\] (in fact, \[\mu_{opt} \approx 2 - \delta\], with \[\delta < 0.1\] small \[10\]) optimal regimes in the \[l_0 - \rho\] space. Along each fitting line, we have a fairly same \[\mu_{opt}\] value. But across them, \[\mu_{opt}\] presents a strong variation, from \[2.9\] to \[2.1\] (\[\approx 30\%\] decrease). In foraging, it is an important result. An slight increasing in either the perceptual range or resources availability can considerably change the “movement diffusiveness degree” necessary to enhance search efficiency. Mechanisms which take advantage of such fact in setting search strategies could be an evolutive protection against periods of hard conditions \[11\] (see below).

Likewise interesting is the \[\rho\] and \[l_0\] interplay for the optimal strategy. For larger \[l_0\] the crossover occurs also for larger \[\rho\]. So, for scarcer landscapes (higher \[l_0\]), efficient anomalous strategies arise for a wider range of values of \[\rho\]. \[\beta = \rho_v / l_0\] alone does not define \[\mu_{opt}\] in the crossover region, inset of Fig. 3. Although displaying a small variation, \[\beta\] is not constant along a curve of constant \[\mu_{opt}\]. But for \[\beta\] sufficiently high or low, it becomes the only parameter setting the best strategy.

The frequency of reorientation events should be significantly influenced by \[\rho\] and \[\beta\]. Figure 4 shows \[\Gamma\] versus \[\beta\], for \[\mu = 1, 1.2, 2, 0, 3.0\] and \[l_0 = 100\]. For large \[\beta\] there is a

FIG. 1. (a) The random search dynamics, illustrating the perceptual range \[\rho_v\] and neighbor targets mean distance \[l_0\]. (b) For distinct \[\mu\], search realizations are displayed, with dots indicating the starting positions of the first \(10^2\) steps.
monotonical increase of $\Gamma$. For a fixed $\rho$ a wider perceptual range reduces the necessity of looking at random [rule (2)] and thus the number of reorientations. Figure 4 also shows that the growth rate of $\Gamma$ depends on the step length distribution $p(s)$. Interestingly, the presence of an inflection point, which is absent near the ballistic ($\mu \to 1$) behavior, characterizes a qualitative change in the growth rate of $\Gamma$ towards the $\beta \to 1$ saturation limit, when targets are often within the searcher perceived distance ($r_\eta \to l_0$), and on average they are encountered after only $N_{\text{enc}} \to 1$ step. Since the inflection points always lie in the crossover region neighborhood (numerically verified), they are good indicators of when to switch the search regime.

Figure 5 shows $\Gamma$ versus $\beta$ for different $\rho = l_0^{-2}$. For $\beta \lesssim 0.2$ and fixed $\mu$ (inset of Fig. 5), $\Gamma$ depends basically only on $\beta$, distinct $\rho$ present the same $\Gamma$. It is in this regime of scarcity or small perceptual range that the adoption of the proper search strategy can make the difference between survival or extinction in the context of animal foraging [4]. Moreover, for low $\beta$ we have $\Gamma \propto \beta^v$, with $v = 2.03$ ($v = 1.58$) for $\mu = 3.0$ ($\mu = 2.0$). It has been proved in one dimension [16], but argued to hold in general [11], that $N_{\text{enc}} \sim (\lambda/r_v)^{(\mu-1)/2}$, for $\lambda/r_v \gg 1$, with $\lambda \propto (r, \rho)^{-1}$ the mean free path of the target distribution, hence $\Gamma \sim \beta^{v-1}$, for $\beta \ll 1$ (so, the type of scale in our simulations). Identifying $v = \nu - 1$, a nice agreement comes from $\mu = 3$. The discrepancy for $\mu = 2$ ($\nu = 1.58$ and $\mu = 1 = 1$) should be expected since the analytic expression is for the extended space, in contrast to our periodic boundary conditions (recall for $\mu = 2.0$ the walker will reach the borders more often than for $\mu = 3.0$). These results are consistent with the scale-invariant structures of set of sites visited by a Lévy searcher (Lévy dust) of fractal dimension $\mu - 1$ [14].

The optimization of $\eta \sim \Gamma/(\langle |\ell| \rangle)$ is a compromise between maximizing $\Gamma$ and minimizing $\langle |\ell| \rangle$. In Fig. 4 larger $\Gamma$ for smaller $\mu$ relate to a higher frequency of large step lengths decreasing the number of steps for an encounter. In contrast, smaller mean step lengths $\langle |\ell| \rangle$ result from larger $\mu$. As Figs. 2 and 3 show, the balance between these opposite trends to set up the best strategy strongly depends on the interplay between $r_\eta$ and $l_0$.

Figure 5 indicates $\Gamma$ independent on $\rho$ for $\beta \lesssim 0.2$. We emphasize that distinct efficiencies $\eta$ emerge in landscapes with distinct target densities in Fig. 6 for $l_0 = 50, 75, 100, 150$. This is essentially due to the influence of step truncations by target encounters [rule (2)] on $\langle |\ell| \rangle$, which depends on $l_0$. But such an effect is self-similar in the low $\beta$ regime and $\mu$ fixed: For $\beta \lesssim 0.2$ the scaling $\eta(\beta, l_0) = k(\beta, k_0)$ holds, as exemplified for $\eta(\beta, l_0 = 50)$ in Fig. 6 (a behavior similar to that seen for $\Gamma$ in Fig. 5).

When energetic balance is assumed (incomes versus energetic costs), a second order phase transition separating an active phase (searcher alive) and an absorbing state (death) takes place [11]. In such case, there are instances in which only superdiffusive dynamics leads to survival and the parameters space Brownian portion becomes forbidden. Depending on the absorbing state onset loci, laying in or out the crossover region, conceivably the associated critical phenomenon will belong to distinct universality classes (an issue for future analysis). Although energetic effects are not considered here, clearly the crossover region represents the separation of two different dynamical behaviors. It has been observed [15] that
in a RS, the emergence (landscape induced) of power-law distribution of step lengths can give rise to dynamical phase transitions [17,18]. So, this might be the case here (another issue for future studies), as substantiated by $\Gamma$ in Fig. 4, displaying inflexion points in such region, and by the $\eta$ scale features in Fig. 6.

IV. REMARKS AND CONCLUSION

We have clarified the perceptual range and targets density interplay in RS. The ratio $\beta = r_v/l_0$ significantly impacts the optimal search features and delimits a crossover region separating the regimes with superdiffusive and diffusive efficient search dynamics. At the critical situation [11] of small $\beta$, an scale-invariant behavior of the efficiency and rate of reorientation events emerges. In this case $\beta$ is the major factor defining the proper optimal strategy. For intermediate values, no longer $\beta$ is the sole parameter defining $\mu_{\text{opt}}$. Then, the efficiency difference between distinct strategies is not so drastic (although eventually sufficing to set a selection between specialists, high $r_v$, and generalists, high $l_0$, in the context of evolutive ecology [19]).

Our work is a step towards understanding when “pressure” is so strong to force strategy changes in RS. To illustrate the relevance of such findings, we end mentioning some actual instances of crossover in biological systems.

Selfish genes face the problem of how to interact (by encounters) with other similar cells. So their foraging approach should clearly depend on what exactly they are looking for, setting $r_v$ and $\rho$ and hence the best strategy. The optimal superdiffusive strategy, however, may be hard to achieve, e.g., if individualistic rather than collective organization is followed. This is nicely exemplified in the case of fungal mycelium [20]. Numerical investigations of bacterial growth show that “chemotactic feedback” allows communication, thereby giving rise to cooperative modes of behavior [21]. As an consequence, detection power (our $r_v$) can change, resulting in a translation in the parameter space. It facilitates search improvement since $\mu_{\text{opt}}$ shifts from small to large values. Data on tundra swans [22] indicates they follow optimal foraging theories [4], so that parameter space maps (cf. Fig. 3) are helpful to understand their dynamics. In particular, certain foraging strategies seem to confer higher advantages, e.g., for swans forming pairs. Thus, foraging optimization, when fundamental, drives social organization [23] to enable crossovers. It brings up a phenomenon relevant in spreading processes of agents which act alone, but in synergy [24]. Their foraging strategy may change from exploitative (normal diffusion) to explorative (superdiffusion) depending, respectively, on the positive or negative character of the synergetic effects.

Lastly, we mention that due to its nature, aquatic environments frequently vary their local availability of resources, i.e., $\rho$ [6]. A very impressive recent survey [25] using more then $10^3$ data points for more than a dozen marine species has convincingly shown (see also Ref. [26]) that predators adapt their foraging strategy in a manner consistent with a Lévy search, with $\mu$ “tuned” with $\rho$. Equally striking is the behavior of jellyfish in the wild, which displays movement patterns similar to optimal Lévy strategies, with significant changes of $\mu$ (from 1.2 to 2.9), potentially a response to $\rho$ [27].

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[12] Trapping effects in an eventual back and forth motion between two very close targets is avoided by restricting the number of alternate successive visits to neighbors targets.


