ΛCDM model with dissipative nonextensive viscous dark matter

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**Highlights**
- New interpretation of the bulk viscosity based on the nonextensive framework.
- ΛCDM model plus dissipative dark matter.
- Nonextensive/dissipative correspondence (NexDC).

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**Abstract**

Many models in cosmology typically assume the standard bulk viscosity. We study an alternative interpretation for the origin of the bulk viscosity. Using nonadditive statistics proposed by Tsallis, we propose a bulk viscosity component that can only exist by a nonextensive effect through the nonextensive/dissipative correspondence (NexDC). In this paper, we consider a ΛCDM model for a flat universe with a dissipative nonextensive viscous dark matter component, following the Eckart theory of bulk viscosity, without any perturbative approach. In order to analyze cosmological constraints, we use one of the most recent observations of Type Ia Supernova, baryon acoustic oscillations and cosmic microwave background data.

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1. Introduction

The idea of dissipative phenomena has been recurrently debated in the issues related with cosmology, being bulk viscosity the main effect considered. From a historical viewpoint, the standard theory of bulk viscosity was proposed by Eckart in 1940 [1] and the connection with Cosmology occurred in the 70’s with Weinberg [2,3], Ellis [4] and others [5,6]. As a matter of fact, due the cosmological principle, a single fluid with bulk viscosity could explain both dark matter and accelerated expansion [7–11]. Indeed, as the bulk viscous pressure is negative, this is consistent with the effects usually attributed to dark energy as well as with the current accelerating expansion of the universe [5,12,13]. Furthermore, the so-called Creation of Cold Dark Matter (CCDM) models [14] are similar to bulk viscous cosmological models. Macroscopically, this model is described through of the introduction of a negative pressure due to matter creation, where this mechanism accelerates the universe without considering the dark energy component. From a phenomenological standpoint, the introduction of viscosity in the cosmological fluid should be interpreted as an effect of creation of particles due the non-stationary gravitational field of the expanding universe [15]. More recently, the effect of shear viscosity has also been introduced in the cosmic fluid in order to explain the expansion [16]. The effect of a viscous pressure also could be incorporated within of inflationary scenario, e.g.

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such pressure comes either from the decay of heavy fields in light fields or by production of particles by inflaton [17], the possibility of an attractor for system of equations describing a tachyon warm inflationary model with bulk viscosity [18], the fluid description of the inflationary universe which includes bulk viscosity in the equation of state [19] (See also [20–23], and for a review on the cosmological bulk viscosity, see Ref. [24,25]).

On the other hand, the Boltzmann–Gibbs entropy has been extended in order to capture all properties of complex systems. The Tsallis entropy has been widely investigated in both theoretical and applications context. This entropy is based on the nonadditive statistics and dynamics of relativistic fluid. In Section 3 we will present the main dynamicsequationsofviscousfluid, using an acoustic oscillations and CMB data.

In order to implement a mechanism which relates the nonextensive effect with the cosmological bulk viscosity, we assume three simplifications. First, the NexDC shall be interpreted from way to preserve the extensive/additive limit $q = 1$; Second, by considering the cosmological principle, most of dissipative effects should be ignored, e.g. shear forces, heat conduction and diffusion. Therefore, the only scalar dissipative component which is consistent with the cosmological principle is the bulk viscosity; Third, by assuming a possible deviation from $\Lambda$CDM behavior, we consider that the bulk viscosity is small. In this limit, we can expand the distribution function around $q = 1$ in order to obtain an expression consistent with the first argument. Therefore, taking into account these simplifications, we propose a thermodynamical interpretation for the bulk viscosity based on the nonextensive effect.

In order to test this interpretation we consider an extension of the flat $\Lambda$CDM model with a nonextensive viscous dark matter component, and we test the observational viability using the most recent observations of Type Ia Supernova, baryon acoustics oscillations and CMB data.

This paper is organized as follows. In Section 2 we will make a rapid description on the connections between Tsallis statistics and dynamics of relativistic fluid. In Section 3 we will present the main dynamics equations of viscous fluid, using the nonextensive bulk viscosity in the context of Eckart theory. In Section 4 we use SN Ia, CMB and BAO data to constrain the nonextensive viscous dark matter with observations. The conclusions and discussions are summarized in Section 5.

2. Nonadditive statistics

The Boltzmann–Gibbs entropy has been extended in order to capture some properties of complex systems. In this concern, the Tsallis entropy has been widely investigated in both theoretical and applications context. This entropy is based on the formula [26,36]

$$S_q = k_B \frac{1 - \sum_{i=1}^{W} p_i^q}{(q - 1)} \text{ for } q \neq 1$$

$$= -k_B \sum_{i=1}^{W} p_i \ln p_i \text{ for } q = 1$$

where $k_B$ is the Boltzmann constant and $p_i$ denotes the probabilities of the $i$-th microscopic states, the average runs over the total number of states $W$ and $q$ is the Tsallis nonextensive parameter. Note that for $q = 1$ the classical Boltzmann Gibbs (BG) statistics is recovered and thus a departure of the exponent $q$ from the value 1 signals a departure from BG statistics.

The connection between the Tsallis framework and relativistic thermodynamics has been introduced through the relativistic kinetic theory. The nonextensive energy–momentum tensor is defined via the relativistic $q$-distribution function [33]

$$T_{\mu\nu}^{q\beta}(x) = \frac{c}{(2\pi \hbar)^3} \int \frac{d^3p}{E} p^\nu p^\beta f_q(x, p).$$

where the 4-momentum is $p^\nu = (E/c, \mathbf{p})$ and the energy $E/c = \sqrt{p^2 + m^2c^2}$, with $m$ being the mass of the relativistic particles. The generalized distribution was proposed by using the relativistic Boltzmann equation in the context of the proof of $H$-theorem [34,35,37,38]

$$f(x, p) = \left[1 + (1 - q) (\alpha(x) + \beta_{\nu}(x) p^\nu)\right]^{1/q}$$

with $\alpha(x)$ and $\beta_{\nu}(x)$ being a scalar and a 4-vector, respectively.
3. Dynamics of nonextensive viscous fluid

We consider a new interpretation for the viscous dark matter which assumes the dissipation process as a form of bulk viscosity in the cold dark matter fluid [39]. This viscous model belongs to ΛCDM class, being Λ responsible by accelerated expansion of the universe and cold dark matter (CDM) behaves as a real fluid with bulk viscosity, i.e., the viscous cold dark matter (VCDM). Specifically, we are proposing a mechanism which is consistent with both the extensive limit ($q = 1$) and bulk viscosity as only dissipative term.

From a mathematical standpoint, it is possible through the expansion of distribution function around $q = 1$ in (2). Therefore, using (3) as $f(x, p) = [1 - (1 - q)\frac{\mu u_\mu}{T(x)} (1 - q)]$, the Ref. [33] has shown that

$$f(x, p) \approx f_{q=1}(x, p) + \frac{1}{2}(q - 1)\left(\frac{p^\mu u_\mu}{T}\right)^2 f_{q=1}(x, p)$$

(4)

where $|1 - q| \frac{\mu u_\mu}{T} < 1$ and $|1 - q| \left(\frac{p^\mu u_\mu}{T}\right)^2 < 2$. This calculation provides

$$T^{\alpha\beta}_q = T^{\alpha\beta}_{q=1} + (q - 1)\Delta T^{\alpha\beta}.$$  

(5)

Therefore, assuming the mathematical approach related to (5), it shall be possible to use the mechanism above proposed as well as to implement the interpretation of viscous dark matter based on the microscopic effect related with nonextensive distribution function in (2). It is worth emphasizing the format of the dissipative term, $\Delta T^{\alpha\beta}$, which will follow the Eckart theory of bulk viscosity

$$\Delta T^{\alpha\beta} = \xi (g^{\alpha\beta} - u^\alpha u^\beta) \nabla_\mu u_\mu,$$

(6)

where $u^\alpha$ is the four-velocity, $g^{\alpha\beta}$ is the metric and $\xi$ is the bulk viscosity coefficient. By choosing an arbitrary hydrodynamic frame of reference, whose four-velocity obeys $u^\alpha u_\alpha = 1$. By replacing (6) in (5), we obtain

$$T^{\alpha\beta}_q = (\rho + P_{\text{eff}}) u^\alpha u^\beta - P_{\text{eff}} g^{\alpha\beta}$$

(7)

where $\rho$ is the energy density and the effective pressure reads

$$P_{\text{eff}} = P_k + \Pi,$$

(8)

with $P_k$ the kinetic pressure and $\Pi = -3(q - 1)\xi H$ the bulk viscosity pressure. It is worth emphasizing that the inhomogeneous equation of state (8) is a consequence of the nonextensive effect, i.e., the viscous pressure becomes null in the extensive limit, $q = 1$. However, as the bulk viscosity has also been considered in order to mimic a dark energy equation of state [44], the nonextensive bulk viscosity likely has a role in the context of the dark energy fluid. In this regards, this approach will follow to same class of dark energy models, which has been investigated considering a dark fluid with the inhomogeneous or imperfect equation of state [45,46].

As is well known, the energy conservation in (7) leads to

$$\dot{\rho} + 3H(\rho + P_{\text{eff}}) = 0,$$

(9)

with each component of the cosmic fluid individually conserved. Therefore, the conservation of viscous dark matter component is given by

$$\dot{\rho}_{\text{VCDM}} + 3H(\rho_{\text{VCDM}} + p_{\text{VCDM}}) = 0,$$

(10)

where the viscous dark matter component has only an intrinsic pressure from the bulk viscosity, i.e. $p_{\text{VCDM}} = \Pi$. It is worth emphasizing that the bulk viscosity models has a problem, known as integrated Sachs– Wolfe effect (ISW). Actually, the bulk viscosity is responsible by a large time variation of gravitational potential at late times [47]. In order to minimize this effect, we use a constant bulk viscosity $\xi = \xi_0$, where $\Lambda$ drives the accelerated expansion. By taking into account this issue, we use the bulk viscous pressure as

$$\Pi = -3\xi_0 H,$$

(11)

1 Hereafter the convention that $\hbar = k_b = c = 1$ will be used.

2 Although often studied for decades, considering only first order deviations from equilibrium, it is well known that Eckart theory has been failing at perturbative levels due to an instability in all its equilibrium states and signal propagation with superluminal velocities [39–43]. In this work we did not make any perturbative approach.
where $\xi_0 = (q - 1)\xi_0$. Now, by replacing (11) in (10), this new interpretation of viscous dark matter provides the following continuity equation
\[ \dot{\rho}_{VCDM} + 3H\rho_{VCDM} - 9H^2\xi_0 = 0. \]  
\( \text{(12)} \)

From the dynamical standpoint, a spatially-flat expanding universe follows the Friedmann equation in terms of dimensionless density parameter $\Omega$, given by
\[ H(z) = H_0\left[\Omega_{b,0}(1 + z)^3 + \Omega_{\Lambda}(1 + z)^{\alpha} + \Omega_{r,0}(1 + z)^4\right]^{1/2}, \]
\( \text{(13)} \)
where $\Omega_{b,0}$ is the current baryonic density parameter, $\Omega_{r,0}$ is the current radiation density parameter, $\Omega_{VCDM}(z)$ is the nonextensive viscous dark matter density parameter, $\Omega_{\Lambda}$ is the dark energy density parameter, $z$ is the redshift and $H_0$ is the Hubble constant. By replacing (13) in (12), we obtain
\[ (1 + z)\frac{d\Omega_{VCDM}(z)}{dz} - 3\Omega_{VCDM}(z) + \xi_0[\Omega_{b,0}(1 + z)^3 + \Omega_{\Lambda} + \Omega_{r,0}(1 + z)^4]^{1/2} = 0. \]
\( \text{(14)} \)
Here, we define a dimensionless nonextensive bulk viscosity parameter as $\xi_0 = \frac{24\pi G\rho_0}{H_0^2}$. The effect of the bulk viscosity is typically associated with the nonextensive property of the fluid which has been introduced through the strong statistical correlations among hydrodynamic 4-velocities of the fluid. Microscopically, this nonextensive effect should be captured through the $q$-distribution function (3).

The differential equation solution (14) provides us the evolution of the viscous dark matter density parameter as function of redshift, $\Omega_{VCDM}(z)$. From the numerical standpoint, we use the following values of by $\Lambda$CDM model from WMAP9 results [48] with $\Omega_{b,0}h^2 = 0.02264$ and $\Omega_{c,0}h^2 = 4.18343 \times 10^{-5}$ with $h = H_0/100$ $\text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} = 0.70$. Here we use $\Omega_{VCDM}(0) = 1 - \Omega_{b,0} - \Omega_{\Lambda} - \Omega_{r,0}$. Therefore, taking this in account, we obtain two free parameters, i.e. $\xi_0$ and $\Omega_{\Lambda}$ which shall be constrained through observational data.

The time evolution of the density parameters $\Omega_i$ with $i = b, VCDM, \Lambda$ and $r$ are calculated through the combination of Eqs (12)-(14), where we obtain
\[ \Omega_b(a) = \frac{a^3}{\left(a^{-3} + \frac{\Omega_{VCDM}(0)}{\Omega_{b,0}} + A + Ba^{-4}\right)}, \]
\( \text{(15)} \)
\[ \Omega_{VCDM}(a) = \frac{\frac{\Omega_{b,0}}{\Omega_{VCDM}(0)}a^{-3} + C + \frac{\Omega_{\Lambda,0}}{\Omega_{VCDM}(0)}a^{-4}}{1}, \]
\( \text{(16)} \)
\[ \Omega_{\Lambda}(a) = \frac{A^{-1}a^{-3} + \frac{\Omega_{VCDM}(a)}{\Omega_{\Lambda,0}} + 1 + Da^{-4}}{a^{-4}}, \]
\( \text{(17)} \)
\[ \Omega_r(a) = \frac{B^{-1}a^{-3} + \frac{\Omega_{VCDM}(a)}{\Omega_{r,0}} + D^{-1}a^{-4}}{a^{-4}}, \]
\( \text{(18)} \)
where $A = \Omega_{\Lambda,0}/\Omega_{b,0}$, $B = \Omega_{r,0}/\Omega_{b,0}$, $C = \Omega_{VCDM}(a)/\Omega_{VCDM}(0)$ and $D = \Omega_{r,0}/\Omega_{\Lambda,0}$, where we assume $\Omega_{b,0}h^2 = 0.02264$, $\Omega_{VCDM}(0)h^2 = 0.1138$, $\Omega_{c,0}h^2 = 4.18343 \times 10^{-5}$ and $\Omega_{\Lambda,0} = 0.721 - \Omega_{r,0}$ with $h = 0.70$ from WMAP9 results [48].

As result, we show in Fig. 1 the evolution of each component for two selected values of the dimensionless bulk viscosity parameter $\xi_0$.

### 4. DATA ANALYSIS

In this section we derive the constraints with three different cosmological tests. We use observations of 31 binned dataset of Type Ia Supernova from Joint Light-curves Analysis (JLA) dataset found in Table F.1 of Ref. [49], a combined dataset with six observations of baryon acoustic oscillations in the WiggleZ Survey [50] and CMB measures of WMAP [51].

#### 4.1. SN Ia distance modulus

For this cosmological test, the main quantity is the distance modulus. The theoretical distance modulus is given by
\[ \mu(z) = 5 \log[d_L(z)] + 42.384 - 5 \log[h]. \]
\( \text{(19)} \)
where $h = H_0/100$ $\text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ and $d_L$ is the dimensionless luminosity distance, assuming flat geometry
\[ d_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z')/H_0}. \]
\( \text{(20)} \)

In the context of JLA sample [49], the $j$th SN Ia distance modulus is given by
\[ \mu_j = m_{R,j}^\ast - M_B + \alpha x_{1,j} - \beta c_j, \]
\( \text{(21)} \)
Fig. 1. The evolution of the $\Omega_\chi(a), \Omega_{\text{CDM}}(a), \Omega_\Lambda(a)$ and $\Omega_{\text{a}}(a)$ with the log of the scale factor $a$. In the top panel, we plot evolution for the dimensionless nonextensive bulk viscosity parameter, $\xi_0 = 0.2$, whereas in the inferior panel, we use $\xi_0 = 0.01$.

where SN Ia color is given by the parameter $c$, $x_1$ is related to the stretch of the light curve, $m_b^*$ is the peak rest-frame magnitude in the B band and $M_b$ is related to the dependence of the magnitude with the host galaxy

$$M_b = \begin{cases} M^1_b & \text{if } M_{\text{stellar}} < 10^{10} M_\odot \\ M^1_b + \Delta M & \text{otherwise} \end{cases}$$

(22)

where $M_{\text{stellar}}$ is the host galaxy stellar mass. In Ref. [49], an approximation for the distance modulus by a piecewise linear function of $\log(z)$ has been used, i.e, for each segment $z_b \leq z \leq z_{b+1}$ written as

$$\overline{\mu}(z) = (1 - \alpha) \mu_b + \alpha \mu_{b+1},$$

(23)

where $\alpha = \log(z/z_b)/\log(z_{b+1}/z_b)$ and $\mu_b$ is the distance modulus at $z_b$. Therefore, they used a 31 distance modulus binned dataset sample, which is fitted to the measure Hubble diagram by minimizing the $\chi^2$ function

$$\chi^2 = (\mu - \overline{\mu}(z)) C^{-1}(\mu - \overline{\mu}(z)).$$

(24)

For each bin, the parameters $\alpha$, $\beta$, $\Delta M$ and $\mu_b$ are determined (they used 31 log-spaced points $z_b$ in the redshift range $0.01 < z < 1.3$), for the parameter $M^1_b$, has been used a fiducial value of $M^1_b = -19.5$. Through this procedure, has been found a $31 \times 31$ covariance matrix located in the table F.2 of Ref. [49]. For statistical analysis, we are using this covariance matrix to compute the $\chi^2$ function to the binned distances

$$\chi^2_{SN} = (\mu_b(z_i) - \mathcal{M} - 5 \log[d_L(z_i)]) + 42.384)^T C_b^{-1}(\mu_b(z_i) - \mathcal{M} - 5 \log[d_L(z_i)]) + 42.384,$$

(25)

where $\mu_b(z_i)$ is the binned value of distance modulus at redshift $z_i$ (see table F.1 of [49]), and $C_b^{-1}$ its the inverse covariance matrix (see table F.2 of [49]).

For the distance modulus, $\mathcal{M} = M - 5 \log(h)$ is a noise parameter and we need to marginalize, here we use an uniform prior.

Following the same procedure in [52], the marginalized $\chi^2_{SN}$ is given by

$$L_{SN,\text{marg}} \propto \int \exp\left(-\frac{1}{2} \chi^2_{SN}\right) d\mathcal{M} \propto \int \exp\left[-\frac{1}{2} (A - 2 \mathcal{M} B + \mathcal{M}^2 C) d\mathcal{M}\right],$$

(26)

where

$$A = (\mu_b(z_i) - 5 \log[d_L(z_i)]) + 42.384)^T C_b^{-1}(\mu_b(z_i) - 5 \log[d_L(z_i)] + 42.384))$$

$$B = (\mu_b(z_i) - 5 \log[d_L(z_i)]) + 42.384)^T C_b^{-1} 1$$

$$C = 1^T C_b^{-1} 1.$$

Solving that integral we get

$$\chi^2_{SN,\text{marg}} = A - \frac{B^2}{C}.$$  

(27)
4.2. BAO parameter

For this cosmological test, we use the BAO parameter \( A(z) \) and the distortion parameter \( F(z) \) given by the WiggleZ Survey [50] as the observational constraints, \( c = 1 \).

\[
A(z) = D_V(z) \left( \frac{\Omega_m H_0^2}{m^2} \right)^{1/2},
\]

where

\[
D_V(z) = \left[ d_s^2(z) \frac{z}{H(z)} \right]^{1/3},
\]

\[
F(z) = d_s(z) H(z),
\]

\( D_V(z) \) is the dilution scale, \( z \) is the redshift, \( \Omega_m \) is the dimensionless matter density parameter and \( d_s(z) \) is the comoving angular-diameter distance, these observables is measured in three redshift bins \( (z_1, z_2, z_3) = (0.44, 0.60, 0.73) \) so that, we have a array of six data values [50]

\[
Y_{obs} = \langle A_1, A_2, A_3, F_1, F_2, F_3 \rangle = (0.474, 0.442, 0.424, 0.482, 0.650, 0.865).
\]

For statistical analysis we performed the usual \( \chi^2 \) function using the respective covariance matrix (see Table 2 of [50])

\[
C = 10^{-3}
\begin{pmatrix}
1.156 & 0.211 & 0.000 & 0.400 & 0.234 & 0.000 \\
0.211 & 0.400 & 0.189 & 0.118 & 0.276 & 0.336 \\
0.000 & 0.189 & 0.441 & 0.000 & 0.167 & 0.399 \\
0.400 & 0.118 & 0.000 & 2.401 & 1.350 & 0.000 \\
0.234 & 0.276 & 0.167 & 1.350 & 2.809 & 1.934 \\
0.000 & 0.336 & 0.399 & 0.000 & 1.934 & 5.239
\end{pmatrix}
\]

\[
\chi^2_{BAO} = Y^T C^{-1} Y,
\]

where \( Y = (Y_{obs} - Y_{theo}) \).

4.3. First peak of CMB

For the CMB analysis, we are going to perform the first peak of CMB data constraint, \( l_1 \). The key quantity for this test is the angular scale of the sound horizon at last scattering which is defined by

\[
l_A = \pi \frac{r(z^*)}{r_s(z^*)},
\]

where \( r(z^*) \) is the comoving distance of last scattering, and \( z^* \) is its redshift

\[
r(z^*) = \frac{c}{H_0} \int_0^{z^*} \frac{dz}{E(z)}
\]

with \( E(z) = H(z)/H_0 \) and \( r_s(z^*) \) is the comoving sound horizon at last scattering

\[
r_s(z^*) = \frac{c}{H_0} \int_{z^*}^{\infty} \frac{c_s(z)dz}{E(z)},
\]

where the sound velocity is \( c_s = 1/\sqrt{3(1 + R_b/(1 + z))} \) with \( R_b = \frac{3}{4} \frac{\Omega_b h^2}{\Omega_m h^2} = 31500 \Omega_b h^2 (T_0/2.7K)^{-4} \). Here, we take \( T_0 = 2.7255 \). The redshift \( z^* \) is obtained using \( \Lambda \)CDM model by means of the following fitting formula [53].

\[
z^* \approx 1048(1 + 0.00124 \omega_b^{-0.738})(1 + c_1 \omega_m^{c_2}),
\]

\[
c_1 = \frac{0.0783 \omega_b^{-0.238}}{(1 + 39.5 \omega_b^{0.763})},
\]

\[
c_2 = \frac{0.56}{(1 + 21.1 \omega_b^{1.81})},
\]

where \( \omega_b = \Omega_b h^2 \) and \( \omega_m = \Omega_m h^2 \).

In order to find \( z^* \), we use the best fit values of WMAP 9 (see Table 4 of [48]), with \( h = 0.70, \Omega_b h^2 = 0.02264 \) and \( \Omega_m h^2 = 0.13644 \) as a priors to fit \( z^* = 1090.96 \).

The position of the first peak, \( l_1 \), is given by [54]

\[
l_1 = l_0(1 - \delta_1),
\]
Fig. 2. 1σ and 2σ confidence regions for the nonextensive viscous dark matter from a joint analysis with SN+BAO+CMB data. $\xi_q$ is dimensionless nonextensive bulk viscosity parameter and $\Omega_\Lambda$ is the dark energy density parameter. The best fit is $\xi_q = 0.0$ and $\Omega_\Lambda = 0.68$.

where

$$\delta_1 = 0.267 \left( \frac{r}{0.3} \right)^{0.1},$$

with $r = \frac{\rho(z^*)}{\rho_m(z^*)}$, which for our model in particular is

$$r = \frac{\Omega_c(1+z^*)^4}{\xi_b(1+z^*)^3 + \Omega_{\text{CDM}}(z^*)}.$$  

From the above results, we use to constraint data with the 1σ observed first peak from WMAP [51], i.e., $l_1 = 220.8 \pm 0.7$. For statistical analysis we performed the usual $\chi^2$ function for each pair ($\xi_q$, $\Omega_\Lambda$)

$$\chi^2_{\text{CMB}} = \frac{(l_{\text{obs}} - l_{\text{theo}})^2}{\sigma_{l_1}^2}.$$  

For the joint analysis we performed

$$\chi^2_{\text{total}} = \chi^2_{\text{SN, marg}} + \chi^2_{\text{BAO}} + \chi^2_{\text{CMB}}.$$  

5. Conclusions and discussions

The Fig. 2 shows the parametric space of the $\xi_q - \Omega_\Lambda$ plane for our joint analysis, SN+BAO+CMB, with 68% and 95% confidence level, where $\xi_q$ is the nonextensive bulk viscosity parameter and $\Omega_\Lambda$ is the dark energy density parameter. The best fit is $\xi_q = 0.0$ and $\Omega_\Lambda = 0.68$, i.e. the $\Lambda$CDM model. However, we obtain a 2σ estimate upon the nonextensive bulk viscosity parameter given by $\xi_q \sim 0.17$, which should be equivalent to $\xi_{00} \propto 10^6 \text{Pa} \cdot \text{s}$ in SI unity.

In order to investigate the role of the nonextensive effect on the cosmological distances, Fig. 3 shows the SZE/X-ray determined distances for 18 clusters [55] as a function of redshift for a fixed value of $\Omega_M = 0.28$ and for selected values of the bulk viscosity parameter $\xi_q$. As the cosmological distance increases with the bulk viscosity parameter, this effect corresponds to phantom dark energy regime (See [56], for a similar analysis in another context).

A new physical interpretation of the cosmological bulk viscosity has been presented by using a plausible mechanism in the context of nonextensive/dissipative correspondence (NexDC) [33]. As a test, we have used the $\Lambda_{\text{CDM}}$ model for a flat universe in order to investigate dissipative properties for cold dark matter, where cold dark matter has a bulk viscosity pressure, whose nature is interpreted through nonextensive Tsallis statistics. The motivation follows of mechanism which is consistent the extensive limit, bulk viscosity in the context of the NexDC and tiny deviation from the standard framework based on the Maxwell–Boltzmann–Juttner statistics.

From the observational standpoint, we performed a joint analysis from SN Ia, BAO parameters and first peak of CMB. We obtain nonextensive effect only in 2σ estimate, the BAO and CMB tests are crucial to find a upper limit to the nonextensive bulk viscosity parameter $\xi_q$, where in 2σ is $\xi_q \sim 0.17$ or $\xi_{00} \propto 10^6 \text{Pa} \cdot \text{s}$ in SI unity. This result is in agreement to
Fig. 3. SZE/X-ray determined distances for 18 clusters as a function of redshift for a fixed value of $\Omega_M = 0.28$ and selected values of the dimensionless bulk viscosity parameter $\xi_q$.

the previous works which have used the standard interpretation for bulk viscosity \([39,57–60]\), i.e., $\xi_0$ belongs to interval $10^4 \text{Pa} \cdot \text{s} \leq \xi_0 \leq 10^7 \text{Pa} \cdot \text{s}$. It is worth mentioning that the best fit points towards LCDM model, in accordance with the observations.

Last but not least, we emphasize the possibility of the microscopic nonextensive approach be able to describe the dark energy through the inhomogeneous equation of state \([8]\). Thus, this description can be investigated in the context of cosmography \([61]\) and in the quintessence scenarios through the equivalence between the bulk viscosity and the time-dependent equation of state \([62]\). These issues will appear in a forthcoming communication.

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